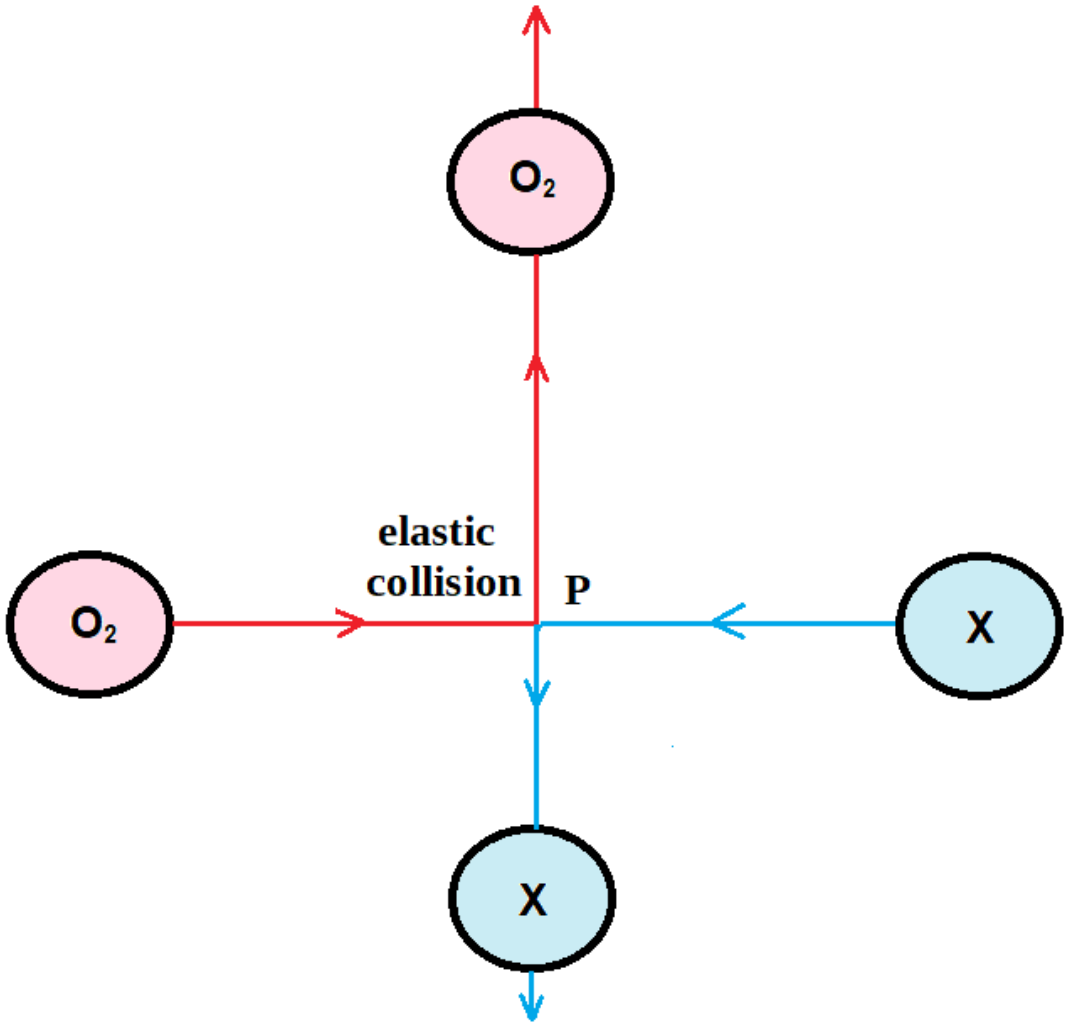
A Problem in Kinetic Theory of Gases



Surface of the Earth

An oxygen particle O_2 (M = 32 kg/kmol) collides elastically with an unknown particle X at point P, very near the surface of the Earth, as shown in the diagram. The temperature is 0 °C. After the collision the oxygen particle goes straight up. If the O_2 particle does not collide with any other particle in its way up, how high it will go? The Boltzmann's constant is 1.38 E-23 J/°K.

Solution

At the beginning the kinetic energy of the oxygen particle is:

$$KE = \tfrac{1}{2}\cdot m_o \cdot v^2_{rms} = 3/2 \cdot k \cdot T$$

Where: KE = kinetic energy of the oxygen molecule

m_o = mass = $(0.032\ kg/6.02\times10^{23}) = 5.32\times10^{-26}\ kg$

v_{rms} = root mean squared speed of the oxygen particle

k = Boltzmann's constant

T = absolute temperature

After the collision, the KE of the oxygen particle gradually changes to GPE:

$$3/2 \cdot k \cdot T = m_o \cdot g \cdot h$$

Where h is the maximum height above the surface of the Earth achieved by the oxygen particle. Solving for h:

$$h = \frac{3 \cdot k \cdot T}{2 \cdot m_o \cdot g} = \frac{(3)(1.38\times10^{-23}\ J/°K)(273\ °K)}{(2)(5.32\times10^{-26}\ kg)(9.8\ m/s^2)}$$

h = 10,800 m = 10.8 km