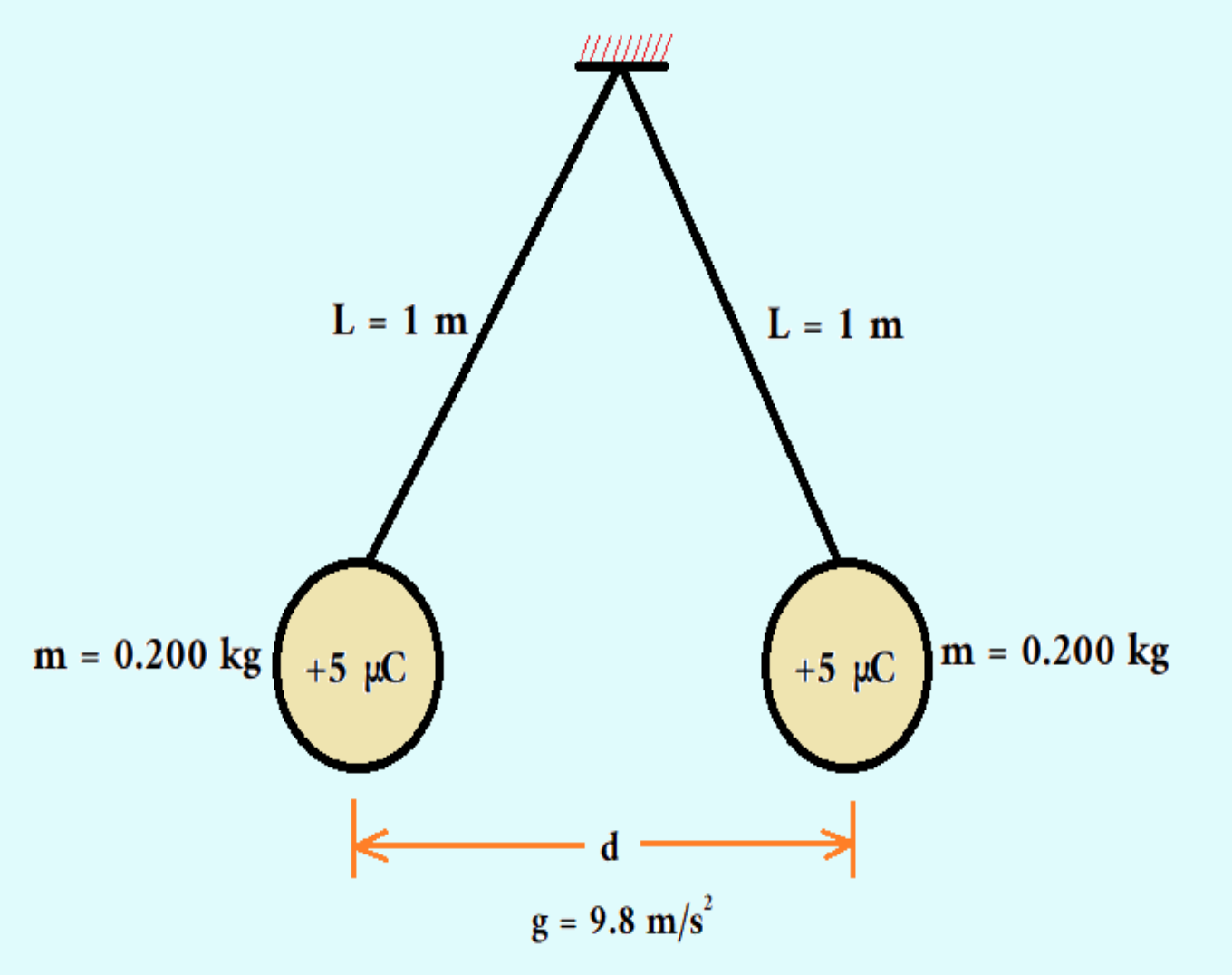
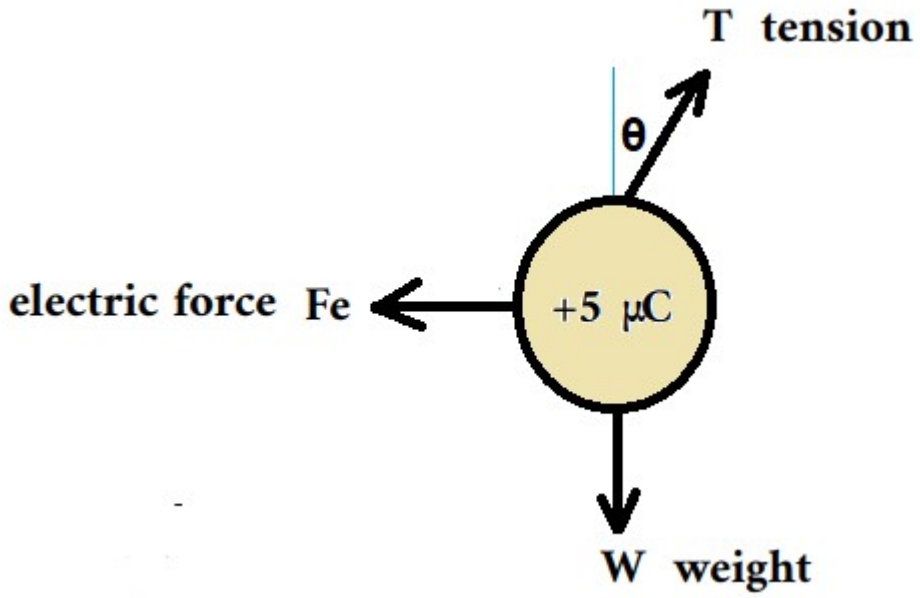
A Problem in Mechanics and Electric Charges



Two 0.200 kg spheres are connected to light strings of length 1 m and then allowed to hang from the ceiling as shown in the diagram. They separate from each other after every sphere is given a charge of +5μC. If gravity is 9.8 m/s^2, what should be the separation d between the two spheres?

SOLUTION

There are three forces acting on each sphere: the tension T, the weight W, and the electric force Fe. The diagram below shows the forces acting on the left sphere. The angle θ is the angle that tension makes with the vertical.



A balance of horizontal forces gives: $T \cdot \sin\theta = Fe$ **[Equation 1]**

A balance of vertical forces gives: $T \cdot \cos\theta = W$ **[Equation 2]**

The separation d between the two spheres is 2 x 1 m x sin θ = 2 sin θ

$$Fe = K \cdot Q_1 \cdot Q_2 / d^2 = (9 \times 10^9 \text{ N.m}^2/\text{C}^2)(5 \times 10^{-6} \text{ C})(5 \times 10^{-6} \text{ C})/(2 \sin\theta)^2$$

$$Fe = 0.05625 / (\sin\theta)^2$$

$$W = m \cdot g = 0.2 \text{ kg} \times 9.8 \text{ m/s}^2 = 1.96 \text{ N}$$

Dividing equation 1 by equation 2 we obtain:

$$\tan \theta = Fe/W$$

$$\tan \theta = 0.05625 /(\sin \theta)^2 \ (1.96 \text{ N}) = 0.0287 / (\sin \theta)^2$$

$$\tan \theta \times (\sin \theta)^2 = 0.0287$$

$$(\sin \theta)^3/\cos \theta = 0.0287$$

By using trig identities or by try and error substituting different angles, solving the equation above will give $\theta = 17.5^o$

The separation d is: $2 \times 1 \text{ m} \times \sin 17.5^o = 0.60 \text{ m}$

Conclusion: The separation d between the two spheres is 0.60 m