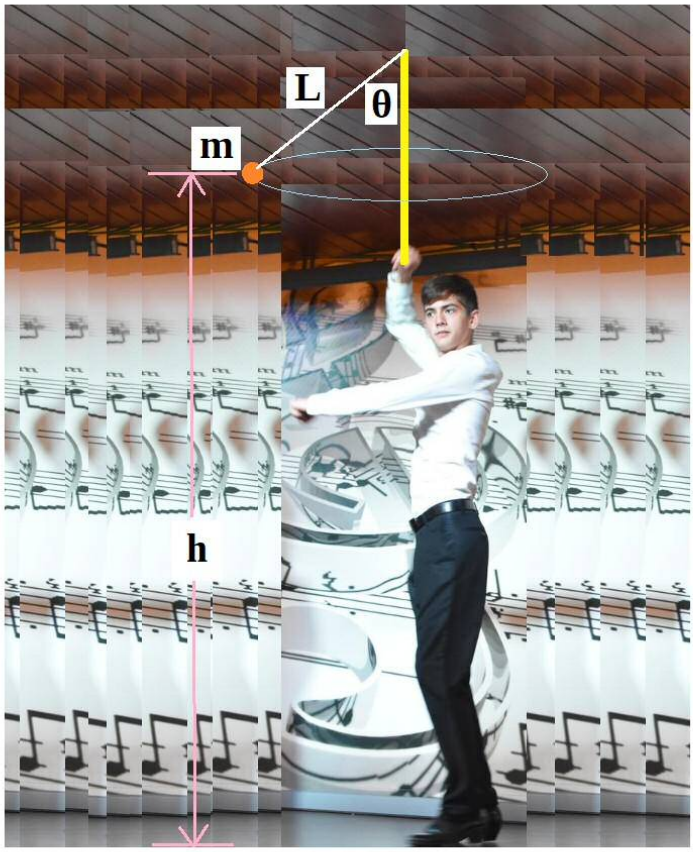
ROTATION IN A HORIZONTAL PLANE



A ball of mass m on a string of length L is rotating in a horizontal circle as shown in the picture. The plane of the circle has a height h from the floor and the string makes an angle θ with the vertical. Gravity is g. If suddenly the string breaks, what horizontal distance does the ball travel until it hits the floor? Express your answer in terms of m, L, θ, h, and g.

SOLUTION

The radius of the circle is $L \cdot \sin\theta$
To find the horizontal distance traveled, or range, we need to know the speed of the ball when it became a projectile, and the time it took to strike the ground. If we do a balance of forces on the ball before the string breaks:

$\sum F_x = 0$: $T \cdot \sin\theta = mv^2/L \cdot \sin\theta$ [Equation 1]

$\sum F_y = 0$: $T \cdot \cos\theta = m \cdot g$ [Equation 2]

Let's divide Equation 1 by Equation 2:

$$\tan\theta = v^2/(L \cdot g \cdot \sin\theta)$$

Solving for v: $v = \sqrt{L \cdot g \cdot \sin\theta \cdot \tan\theta}$

Now we find the time t it takes the ball to strike the floor:

$$h = (½) \cdot g \cdot t^2$$

Solving for t:

$$t = \sqrt{2 \cdot h/g}$$

The horizontal distance traveled d or range is:

$$d = v \cdot t = \sqrt{L \cdot g \cdot \sin\theta \cdot \tan\theta \cdot 2 \cdot h/g} = \sqrt{2 \cdot h \cdot L \cdot \sin\theta \cdot \tan\theta}$$

$$d = \sin\theta \sqrt{2 \cdot h \cdot L/\cos\theta}$$